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Algorithms and Probability **Session Sheet P1** **FS 2026**

(For the exercise sessions on 19 February 2026.)

Exercise P1.1 – *Connectivity*

Let $G = (V, E)$ be a two connected graph.

(a) Let (u, v, w) be a path in G of length 2. Show that we can extend this path to a cycle, that is, show that G contains a cycle where u, v, w appear as incident vertices.

(b) Let $e \in E$ be an edge, and $u, v \in V$ be two vertices. Show that there is a path in G from u to v passing through e.
(*Hint:* you may use without proof that in a 2-connected graph, every two edges e, f lie on a common cycle.)

Solution P1.1 – *Connectivity*

(a) Let (u, v, w) be a path in G of length 2. By the 2-connectedness of G, the graph $G[V \setminus v]$ is connected. Hence, there is a u-w path P that does not contain v. Combining P and (u, v, w) gives the desired cycle.

(b) Let $e \in E$ be an edge, and $u, v \in V$ be two vertices. If $e = \{u, v\}$, the statement trivially holds (as e alone forms an u-v path). Otherwise, consider the graph $G' = (V, E \cup \{\{u, v\}\})$ that arises from G by adding the edge $\{u, v\}$ (if it did not exist already anyways). The graph G' is also 2-connected. Applying the hint to the edges e and $\{u, v\}$, we obtain a cycle C containing both e and $\{u, v\}$. Removing $\{u, v\}$ from this cycle gives the desired path.