AnW Kahoot Quiz

Julius (julius.jung@inf.ethz.ch),
Victoria (victoria.miebach@inf.ethz.ch),
Haaroon (hmohammed@ethz.ch),
Hannah Lee (hannahlee.pfennig@inf.ethz.ch),
Rui (rui.zhang@inf.ethz.ch),
Metehan (mkilic@ethz.ch),
Luka (lwedegaertne@ethz.ch),
Timo (herzogti@ethz.ch),
Efe (ealici@ethz.ch),
Ayse (ayssen@athz.ch)

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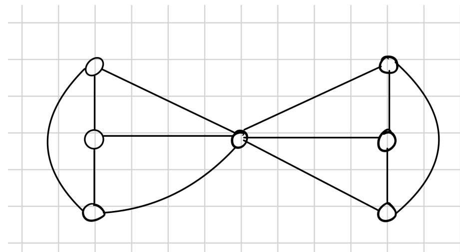
Thank you very much for participating in the Kahoot Quiz, or just checking our Kahoot in general. In this document, you will find some explanation for the solutions. If anything is unclear, feel free to reach out to any of the TAs above.

1 Graph theory

1.1 basic concepts

Question: Let T=(V,E) be a tree with $|V| > 2$, then
Answer:
a) T contains at least two leaves - Yes
b) if $v \in V$ is a leaf then ($V \backslash${v},E) is also a tree - Yes
c) $|E| = |V| - 1$ - Yes
d) T does not contain a cycle - Yes

Question: For any Graph G, the number of vertices of uneven degree must be even.
Answer: Yes, since the sum of all degrees is even.



1.2 Paths

Question: A walk is a path if all edges are visited at most once.
Answer: False. All vertices have to be visited at most once.

Question: For any graph, if every vertex has an even degree, then there exists a Eulercircuit.
Answer: False - The graph has to be connected.

Question: The sum of degrees of all vertices is equal to 2 times the number of edges, for any graph.
Answer: True

Question: Using DFS, we can always find out if there is a path from u to v, for any two vertices u and v.
Answer: True

1.3 Zusammenhang

Question: Every 3-edge-connected graph is also a 3-connected graph?
Answer: False, see the counterexample below which is 3-edge-connected but not 3-connected.

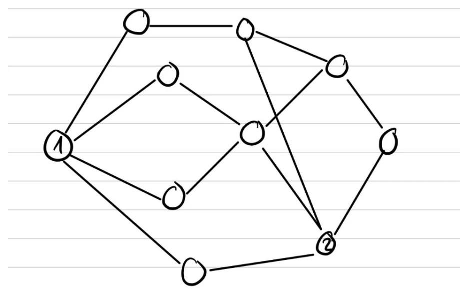
Question: Whats the minimal size of the 1-2 edge-separator and 1-2 separator of the graph below.

Answer: For each 1-2 edge-separator has size at least 4 is the only correct option, for every other there exits one separator with smaller size. And the for each 1-2 separator it has at least a size of 3.

1.4 Kreise

Question: A Graph G contains an Eulerian circuit $\Leftrightarrow$ every vertice degree in G is even.
Answer: False, G also needs to be connected.

Question: Which of these Graphs contains a Hamilton cycle?



1. Every bipartite graph
2. The 13×12 grid graph
3. Every complete graph K_n
4. Every hypercube H_n, for $n > 1$

Answer: 2 and 4 are correct.

1. is incorrect: Not every bipartite graph contains a Hamiltonian cycle (e.g., $K_{2,3}$ does not).
2. is correct: Every grid graph with an even number of vertices contains a Hamilton cycle.
3. is incorrect: The complete graph has to have at least 3 vertices to contain a Hamilton cycle.
4. is correct: Every hypercube H_n for $n > 1$ has a Hamiltonian cycle (constructive proof in the script 1.5.3).

1.5 Matchings

Question: Given the following pseudocode for a matching algorithm on an undirected graph $G = (V, E)$:

$M \leftarrow \emptyset$
while $E \neq \emptyset$ **do**
  pick any edge $e \in E$
  if e contains no vertex already in M:
    $M \leftarrow M \cup \{e\}$
  remove e and all incident edges from E
return M

Which of the following statements is true?

A) It always finds a perfect matching if one exists.

B) It guarantees a cardinality-maximal matching in linear time.

C) It always returns the same matching regardless of edge order.

D) Returns a matching that is at least half the size of a maximum matching.

Answer: D. According to Satz 1.47, the Greedy-Matching algorithm computes an inclusion-maximal matching M_{Greedy} in time $O(|E|)$, which satisfies the following inequality:

$$|M_{\text{Greedy}}| \geq \frac{1}{2}|M_{\max}|,$$

where $M_{\max}$ denotes a cardinality-maximal matching.

Question: If there are no augmenting paths of length 1 with respect to a matching M, then M is inclusion-maximal.
Answer: True. It means there is no edge that can be added to the current matching M without violating the matching property.

Question: *Input: complete graph G with triangle inequality.*

$T \leftarrow \text{MST}(G)$
$O \leftarrow$ vertices with odd degree in T
$M \leftarrow$ perfect matching on $G[O]$
$H \leftarrow T \cup M$
Find Euler tour in H
Construct Hamiltonian cycle using shortcuts
return Hamiltonian cycle

What is the role of the matching step in the 3/2-approximation algorithm for a metric TSP?

A) It is used to eliminate duplicate edges in the MST.

B) It ensures all vertices have even degree for an Euler tour.

C) It replaces the minimum spanning tree with shorter direct edges.

D) It increases the number of edges in the final tour.

Answer: B

Question: Which of the following statements is true for a cardinality-maximal matching M?

A) It contains all vertices of the graph.

B) There is no augmenting path with respect to M.

C) M is always the result of the greedy matching algorithm.

D) Every graph has exactly one cardinality-maximal matching.

Answer: B. According to Berge's Lemma, a matching is cardinality-maximal if and only if there is no augmenting path with respect to it — this makes statement B correct.

Question: The condition in Hall's Theorem is also a necessary condition for non-bipartite graphs. If there exists a set $X \subseteq V$ such that $|X| > |N(X)|$, then there can not be a perfect matching in the graph.
Answer: True. If $|N(X)| < |X|$ for some set X, then there are not enough neighbors to match every vertex in X, hence a perfect matching is impossible.

Question: In a complete graph K_n with the triangle inequality, the Christofides algorithm does not need to compute a matching if n is even.
Answer: False $\rightarrow$ What matters is not the degree of vertices in the original graph, but their degree in the minimum spanning tree (MST). A matching must be computed regardless of whether n is even or odd.

1.6 Colouring

Question: Every tree can be coloured with two colours.
Answer: True, we can colour each even layer of the tree in one and each odd layer of the tree in another colour. Since each edge goes from an odd to an even layer this is a valid colouring.

Question: Let $\Delta(G) = k$ be the maximum degree of each graph G. Which of the following graphs can *always* be colored with k colors?

1. The complete graph
2. Every cycle
3. All connected graphs containing an articulation point
4. None of these graphs can always be colored with k colors

Answer:

1. is incorrect: The complete graph with $\Delta(G) = k$ has k+1 vertices and thus needs k+1 colours.
2. is incorrect: An odd cycle (e.g., C_5) has maximum degree 2 but needs 3 colors.
3. is correct: Graphs with an articulation point can neither be an odd cycle nor a complete graph, the colouring thus follows from Satz von Brooks.

2 Probability theory

2.1 basic concepts

Question: The probability of an event is defined as $Pr[E] := \sum_{\omega \in E} Pr[\omega]$
Answer: True, definition of event, see script: Definition 2.1 page 87

Question: The sample space of a dice throw and a subsequent coin toss can be defined as

a {K,Z} × {1,2,3,4,5,6}

b {K,Z} ∪ {1,2,3,4,5,6}

c {1,2,3,4,5,6} × {K,Z}

d {1,2,3,4,5,6,K,Z}

Answer: c, each atomic event of the probability set consists of both results, the first result the dice toss, the second result the coin toss.

Question: Let A, B, C be three events in a probability space with
$\Pr[A] > 0,\ \Pr[B] > 0,\ \Pr[C] > 0$:

a If $\Pr[A \cup B] = \Pr[A] + \Pr[B]$ then $\Pr[A \cap B] = 0$

b If $\Pr[A] \leq \Pr[B]$ then $\Pr[A \cup C] \leq \Pr[B \cup C]$

c $\Pr[\overline{A}] < 1$

d If $\Pr[A] \leq \Pr[B]$ then $\Pr[A \cap C] \leq \Pr[B \cap C]$

Answer: a, c

2.2 Conditional Probability

Question: $Pr[A|B] = \frac{Pr[B|A] * Pr[A]}{Pr[B]}$
Answer: True

Question: It always holds, that $Pr[A|B] \leq Pr[A]$ if $Pr[B] > 0$
Answer: False

Question: If $A \subset B$ and $Pr[A] > 0$ then $Pr[B|A] = 1$
Answer: True

Question: For any B and $A_1, ..., A_n$ it holds that:
$Pr[B] = Pr[B|A_1] * Pr[A_1] + ... + Pr[B|A_n] * Pr[A_n]$
Answer: False. $A_1, ..., A_n$ have to be mutually disjoint and collectively exhaustive.

2.3 Independence

Question: Let A, B and C be three independent events. Then the events $A \cup B$ and C are also independent.
Answer: True. We can prove this as following:
$Pr[A \cup B | C] = \frac{Pr[(A \cup B) \cap C]}{Pr[C]} = \frac{Pr[(A \cap C) \cup (B \cap C)]}{Pr[C]} = \frac{Pr[A \cap C] + Pr[B \cap C] - Pr[A \cap B \cap C]}{Pr[C]} = \frac{Pr[A] \cdot Pr[C] + Pr[B] \cdot Pr[C] - Pr[A] \cdot Pr[B] \cdot Pr[C]}{Pr[C]} = Pr[A \cup B]$

Question: The events $A, B \subseteq \Omega$ are independent. Are the events A, B and Ω independent, too?
Answer: Yes, all events are independent of Ω.
$Pr[A \cap \Omega] = Pr[A] \cdot Pr[\Omega] = Pr[A]$ (analogous for B)
$Pr[A \cap B \cap \Omega] = Pr[A \cap B] = Pr[A] \cdot Pr[B] \cdot 1 = Pr[A] \cdot Pr[B] \cdot Pr[C]$

Question: If $Pr[A|B] = Pr[A|C]$, then the events B and C cannot be independent.
Answer: False, consider the following counterexample:
$\Omega = A = \{1, 2, 3, 4, 5, 6\}, B = \{1, 2, 3\}, C = \{3, 4\}$
$Pr[B \cap C] = \frac{1}{6} = Pr[B] \cdot Pr[C]$

2.4 Zufallsvariablen

Question: Let X be a random variable with $W_x \subseteq \mathbb{N}_0$ on probability space Ω. Which of the following is not equal to $\mathbb{E}[X]$?

a $\sum_{x \in W_x} x \cdot Pr[X = x]$

b $\sum_{i=1}^{\infty} F_X(i) \cdot Pr[X = i]$

c $\sum_{i=1}^{\infty} Pr[X \geq i]$

d $\sum_{\omega \in \Omega} X(\omega) \cdot Pr[\omega]$

Answer: Correct answer is **b**. All other definitions are included in the script. (**a**: Def. 2.27., **c**: Satz 2.30., **d**: Lemma 2.29.) A counter-example for **b** can be the following setting: Let X denote our reward in a game of a fair coin toss. Heads, we receive CHF 0, tails we receive CHF 10. $\mathbb{E}[X] = 5$ but **b** yields $F_X(10) \cdot Pr[X = 10] = 1 \cdot \frac{1}{2} = \frac{1}{2}$. Because $Pr[X = x]$ is 0 for $x \notin \{0, 10\}$ and we start the sum from 1.

Question: Let X and Y be random variables for the same probability space Ω, such that for all $\omega \in \Omega$ the following holds: $X(\omega) \leq Y(\omega)$. Select the correct options.

a $\mathbb{E}[2X] \geq \mathbb{E}[Y]$

b $\mathbb{E}[XY] \leq E[Y^2]$

c $\mathbb{E}[X - Y] \leq 0$

d $\mathbb{E}[X^2] - \mathbb{E}[Y]^2 + \mathbb{E}[Y^2] - \mathbb{E}[X]^2 \geq 0$

Answer: Correct options are **c** and **d**. **a** is incorrect because we do not know the exact relation between $X(\omega)$ and $Y(\omega)$. A counterexample that satisfies the constraint in the question but not **a** is $\forall \omega \in \Omega. X(\omega) = -10, Y(\omega) = -1$ then $\mathbb{E}[2X] \leq \mathbb{E}[Y]$. This is also a counterexample for **b**. **c** is correct:

$$\begin{aligned}
\mathbb{E}[X - Y] &= \mathbb{E}[X] - \mathbb{E}[Y] && \text{linearity of } \mathbb{E} \\
&= \sum_{\omega \in \Omega} X(\omega) Pr[\omega] - \sum_{\omega \in \Omega} Y(\omega) Pr[\omega] && \text{definition of } \mathbb{E} \\
&= \sum_{\omega \in \Omega} \big(X(\omega) - Y(\omega)\big) Pr[\omega] && \text{simple arithmetic} \\
&\leq 0 && X(\omega) \leq Y(\omega) \text{ and } 0 \leq Pr[\omega]
\end{aligned}$$

d is correct since the given expression is equal to $Var[X] + Var[Y]$ and the variance of a random variable is always non negative.

Question: Let $G = (V, E)$ be a graph with $|V| = n$ and $|E| = m$. We consider Laplace model on $\Omega = \{S \mid S \subseteq V\}$ and the random variable $X =$ #edges over the cut $(S, V \setminus S)$. Which of the following proves that there exists a cut of size at least $\frac{m}{2}$?

a $\mathbb{E}[X] = \frac{m}{4}$

b $Var[X] = \frac{m}{8}$

c $\mathbb{E}[X] = \frac{m}{2}$

d $Var[X] = \frac{m}{16}$

Answer: **c**. Because **c** implies that there exists some elementary event ω such that $X(\omega) \geq \frac{m}{2}$. This implies that there exists a set $S \subseteq V$ such that the cut $(S, V \setminus S)$ includes at least $\frac{m}{2}$ edges. In fact it can be proven that indeed $\mathbb{E}[X] = \frac{m}{2}$ in general.

2.5 Wichtige Diskrete Verteilungen

Question: Let X and Y be random variables with $X \sim Bin(n, p)$ and $Y \sim Bin(m, p)$. How is the random variable $Z = X + Y$ distributed?
Answer: $Z \sim Bin(n + m, p)$. We can see this if we write X and Y as sum of Bernoulli-distributed random variables. It holds: $X = B_1 + ... + B_n$ and $Y = B_{n+1} + ... + B_{n+m}$ where $B_i \sim Bernoulli(p)$ for $i \in \{1, ..., n + m\}$. Since all B_i independent, write $Z = \sum_{i=1}^{n+m} B_i$ to get $Z \sim Bin(n + m, p)$.

Question: Bob flips a coin until he sees tails for the second time. Let $X :=$ number of coin flips. How is X distributed?

Answer: This is known as "Warten auf n-ten Erfolg" or negative binomial distribution.

Question: We think of a different version of Coupon Collector. Alice has now two options:

A: Alice collects the first half of the coupons and rest will be provided.

B: First half of the coupons are provided, Alice collects the remaining half.

Does any of the options have an advantage over the other one?

Answer: If Alice chooses the option A, she is expected to need less coupons to collect them all.

To prove this formally, remember $X_i :=$ Number of coupons needed in phase i.

For the option A: $\mathbb{E}[X] = \sum_{i=1}^{n/2} \mathbb{E}[X_i] = \sum_{i=1}^{n/2} \frac{n}{n-i+1} = n \cdot \sum_{i=\frac{n}{2}+1}^{n} \frac{1}{i} = n \cdot (H_n - H_{n/2}) = n \cdot (\ln n + O(1) - \ln(n/2) - O(1)) = O(n)$.

For the option B: $\mathbb{E}[X] = \sum_{i=\frac{n}{2}+1}^{n} \mathbb{E}[X_i] = \sum_{i=\frac{n}{2}+1}^{n} \frac{n}{n-i+1} = n \cdot \sum_{i=1}^{n/2} \frac{1}{i} = n \cdot H_{n/2} = O(n \ln n)$

2.6 Mehrere Zufallsvariablen

Question: Two random variables X and Y are independent if:

A) $\mathbb{E}[X + Y] = \mathbb{E}[X] + \mathbb{E}[Y]$

B) $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$

C) $\mathbb{P}[X = x \wedge Y = y] = \mathbb{P}[X = x] \cdot \mathbb{P}[Y = y] \quad \text{for all } x \in W_X, y \in W_Y$

D) $\mathbb{E}[XY] = \text{Var}(X) + \text{Var}(Y)$

Answer: C.

Question: You roll a fair 6-sided die until you roll a number divisible by 3. Let X be the number of rolls needed. Then you flip a fair coin X times. Let Y be the number of heads. What is $\mathbb{E}[Y]$?

A) 1

B) 1.5

C) 2

D) 3

Answer: B) 1.5

- Probability of rolling a number divisible by 3 is $\frac{2}{6} = \frac{1}{3}$.
- So $X \sim \text{Geometric}(\frac{1}{3})$, and $\mathbb{E}[X] = \frac{1}{p} = 3$.
- Given X, the number of heads $Y \sim \text{Bin}(X, \frac{1}{2})$.
- Wald's identity gives $\mathbb{E}[Y] = \mathbb{E}[X] \cdot \frac{1}{2} = 3 \cdot \frac{1}{2} = 1.5$.

2.7 Abschätzen von Wahrscheinlichkeiten

Question: Let X be a random variable and $t \in \mathbb{R}$ with $t > 0$. Which of the following does not hold in general?

a $Pr[|X - \mathbb{E}[X]| \geq t] \leq \frac{Var[X]}{t^2}$

b $Pr[|X - \mathbb{E}[X]| \geq t\sqrt{Var[x]}] \leq \frac{1}{t^2}$

c $Pr[|X - \mathbb{E}[X]| \geq \sqrt{t \cdot Var[x]}] \leq \frac{1}{t}$

d $Pr[|X - \mathbb{E}[X]| \geq t \cdot Var[x]] \leq \frac{1}{t^2 \cdot Var[X]^2}$

Answer: Each option except **d** gives the Chebyshev inequality. The correct form of **d** is $Pr[|X - \mathbb{E}[X]| \geq t \cdot Var[x]] \leq \frac{1}{t^2 \cdot Var[X]}$

Question: Bob is a professional coin flipper at D-INFK and he can adjust the probability of seeing heads with a special flipping technique. He chooses this probability as $\frac{1}{n}$ and then performs $5n$ coin tosses. Let X be a random variable denoting the number of occurrences of heads. Then the following holds

$$Pr[X \geq 50] \leq 2^{-50}$$

Answer: True. We can write $X = X_1 + \cdots + X_{5n}$ as the sum of $5n$ independent Bernouilli random variables where X_i is the indicator variable for the i-th toss being heads. We can apply the Chernoff-Bound. We check $\mathbb{E}[X] = 5n \cdot \frac{1}{n} = 5$ and therefore $t := 50 > 2e\mathbb{E}[X]$. We can apply part (iii) of Chernoff Bounds from script theorem 2.70. $Pr[X \geq t] \leq 2^{-t}$.

2.8 Randomized Algorithms

Question: You can change each Las Vegas algorithm to be a Monte Carlo algorithm.
Answer: Yes, since you can just stop it after some time and return utter nonsense. However the converse is not true at all, since a Las Vegas algorithm tells you, if it did non manage to give an answer. A Monte Carlo algorithm will just give you a wrong answer which can not be distinguished from a right one.

Question: If you found a yes-no-algo with one sided-error and 0.01 success rate in O(n)...
Answer: You also found one in O(n) and 0.99 success rate, since you can just repeat it a constant number of times and that does not change the O-notation.

Question: The primality test algorithm which checks n for divisibility by all (n-2) predecessors is in
Answer: It is obviously in O(n) but it is not linear in the input length, since the input length in binary or decimal is in O(log n) and therefore the runtime is exponential in regard to the input length.

Question: Let $t \in \{prime, notprime\}$ be the result of a simple primality test of a number n based on Fermats Little Theorem. Then it holds that.
Answer: $t = prime \Leftarrow$ n is prime since the theorem holds for every prime number, but also for the Carmicheal numbers, which are not prime, so if you find that a number violates the theorem, you know for sure it is not a prime, but if you find that the theorem holds for the number it might just be a Carmicheal number.

Question: Let $t \in \{duplicate, noduplicate\}$ be the result of a Bloomfilter. Then it holds that:
Answer: $t = duplicate \Leftarrow$ n is a duplicate, since if the element has been seen once the positions it is mapped to are set for sure. But if it has not been seen yet, it still can be that other elements have been mapped to its position and the bloomfilter therefore erroneously assumes it to be a duplicate.

3 Algorithm highlights

3.1 Longest path

Question: What is the longest colored path that I can find in polynomial time?
Answer: It is path to length up to $O(log(n))$, since the runtime of finding a colored path of length k is $O(2^k km)$.

Question: What is the probability, in a k-colored graph, that a path(consisting of k vertices) is k-colored?
Answer: $\frac{k!}{k^k}$, since each of the k vertices can choose one of the k colors we have k^k different possibility of coloring a path, but there are only $k!$ possibilites for a colored path. This follows since we can color the first vertices with k colors and the second with k-1 colors and so forth.

3.2 Flow

Question: A network is defined as a 5-tuple with a directed graph (V, A), a source vertex, a target vertex, as well as a capacity function $c : A \to \mathbb{R}$.
Answer: False. Everything is right, but the capacity function only maps to non-negative reals.

Question: Is the following conservation of flow? $\forall v \in V \setminus \{s, t\}$, we have

$$\sum_{u \in V \text{ s.t. } (u,v) \in A} f(u, v) = \sum_{u \in V \text{ s.t. } (v,u) \in A} f(v, u)$$

Answer: This is correct. For all vertices except for the source and the target, we have to have that flow does not appear nor disappear at that point.

Question: What is the runtime of the Ford-Fulkerson Algorithm for integer networks?
Answer: It is $O(mnU)$, where U is the maximum capacity over all edges in the network N. To understand this intuitively, notice that in each iteration of Ford-Fulkerson, we will find shortest paths, augment them and construct the residual network, which is in $O(m)$. In each iteration, we will increase the flow by at least 1, since we have an integer network. The maximum possible flow is $O(nU)$, since vertex s can have at most $n-1$ outgoing edges with at most U capacity.

Question: We have a bipartite graph $(U \uplus W, E)$ We want to find a maximum matching in this bipartite graph. Is the following the correct network definition given the previous setup for finding bipartite matchings using Flow?

- $V = U \uplus W \uplus \{s, t\}$
- $A = E \cup (U \times \{s\}) \cup (W \times \{t\})$
- $c(e) = 1 \; \forall e \in A$

Figure 1: The vertex and edge set as well as capacity function

Answer: True. This is best explained visually / intuitively: We have a

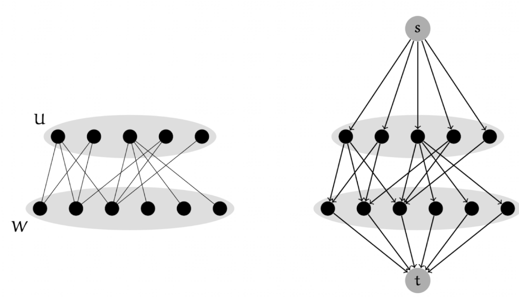


Figure 2: The network construction (right) from a bipartite graph (left). All edges right have capacity 1

network with the same vertices and edges and two new vertices: a source and a target. All edges are directed from the source to the target. If we find the maximum flow in the network on the right, we will force each vertex on side U to only be matched with at most another vertex on side W, since only one unit of flow can pass through to this vertex. At the same time, as long as we can match two vertices $u \in U, w \in W$, there will be one unit of flow from s that can flow to u, then to w, then to t. Therefore, we will find a maximum matching.

Question: Let $N = (V, A, c, s, t)$ be a network with a s-t-path, and let f be a maximum flow in N. If no edge capacity in N is an integer, then f is not integral (*ganzzahlig*).
Answer: True. Proof by contradiction. Assume that f is integral. Due to assumption, there is a s-t-path. Each edge of the path has an non-integer capacity but an integer flow. This implies that each edge can carry more flow. We can then augment the flow using this path, which is a contradiction since f is assumed to be maximum.

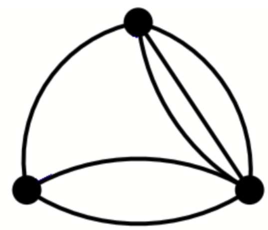
3.3 Smallest circle

Question: Each smallest enclosing circle has 3 points on its edge?
Answer: False, although it normally is 3 it can also be 2, for example consider 3 points where one is exactly in the middle of the other two.

Question: The naive deterministic approach can be improved by randomly selecting the points.
Answer: False, only by selecting the points no better algorithm is achieved, it even gets worse since we have the same expected runtime but now its not even deterministic.

Question: Let p be an element of K, then $out(p, K \setminus \{p\}) =$
Answer: $essential(p, K)$, since if a point is not part of the circle around the set $K \setminus \{p\}$, then the circle around K must be different, i.e. p is essential for the circle.

Question: What does the SmallestCircle algorithm require?
Answer:
1. The algorithm must obviously be able to check which points are contained in the current circle.
2. We duplicate points outside so that they are more likely to be selected. However, if we were to choose uniformly at random between the existing points, this would not be guaranteed in O(n). Instead, we need a way to select the points in O(n) with a probability proportional to their number of copies.
3. We do not need to be able to determine the edge points of a given circle in O(n), but the other way round, we need to determine the circle for given points.
4. We do not need a polynomial algorithm for a few points, we only need an algorithm that terminates for 11 points in bounded time. Since it is never called with more points, we can treat its time as constant, namely O(1).



3.4 MinCut

Question: Let e be an edge and C a minimal cut in the graph G. What holds?
Answer: $(G/e) \geq (G)$, since there could also be another minimal cut in G that does not contain edge e.

Question: What is the probability that $Cut(G)$ the right output for the following graph outputs?

Answer: It's $\frac{1}{2}$, since $(G) = 3$. This means that we can can only contract one of the remaining three edges to get the right result.

Question: By using the concept of bootstrapping, we can reach a runtime of ... for the MinCut algorithm.
Answer: $O(n^2 poly(log(n^2))) = O(n^2 poly(logn))$.